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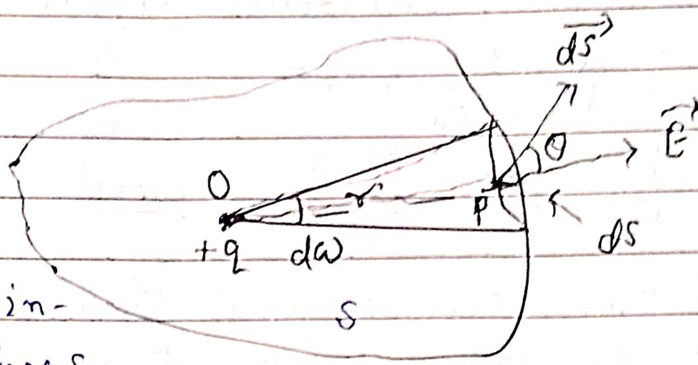
Gauss's law :—

This law gives a relation between the electric flux through any closed hypothetical surface (called a Gaussian surface) and the charge enclosed by the surface. It states that the electric flux ϕ through any closed surface is equal to $1/\epsilon_0$ times the net charge q enclosed by the surface.

$$\therefore \phi = \vec{E}.\vec{dS} = \frac{q}{\epsilon_0}$$

Proof :—



Let us consider a point charge $+q$ situated at O inside a closed surfaces S.

Let $OP = r$. The patch may be represented by a vector $\vec{dS}$ drawn along the normal to the patch area. Let dS be a small patch of area surrounding a point P on the surface.

Let $\vec{E}$ be the electric field intensity at P due to charge $+q$ at O. Its direction is along OP.

The electric flux onward through the patch dS is given by

$$d\phi = \vec{E}.\vec{dS} = E\,dS\cos\theta$$

where θ is the angle between the vectors $\vec{E}$ and $\vec{dS}$.

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\cdot\frac{q}{r^2}$$

$$\therefore d\phi = \frac{1}{4\pi\epsilon_0}\cdot\frac{q}{r^2}\,dS\cos\theta$$

$$= \frac{q}{4\pi\epsilon_0}\cdot\frac{dS\cos\theta}{r^2}$$

By definition, the quantity $\frac{dS\cos\theta}{r^2}$ is the solid angle $d\omega$ subtended by the patch dS at O.

$$\therefore d\phi = \frac{q}{4\pi\epsilon_0} \cdot d\omega$$

The total flux ϕ outward through the surface S is therefore, $\phi = \frac{q}{4\pi\epsilon_0} \oint d\omega$

But $\oint d\omega = 4\pi$, the solid angle subtended by the entire enclosed surface S at O

$$\therefore \phi = \frac{q}{4\pi\epsilon_0} \times 4\pi = \frac{q}{\epsilon_0}$$

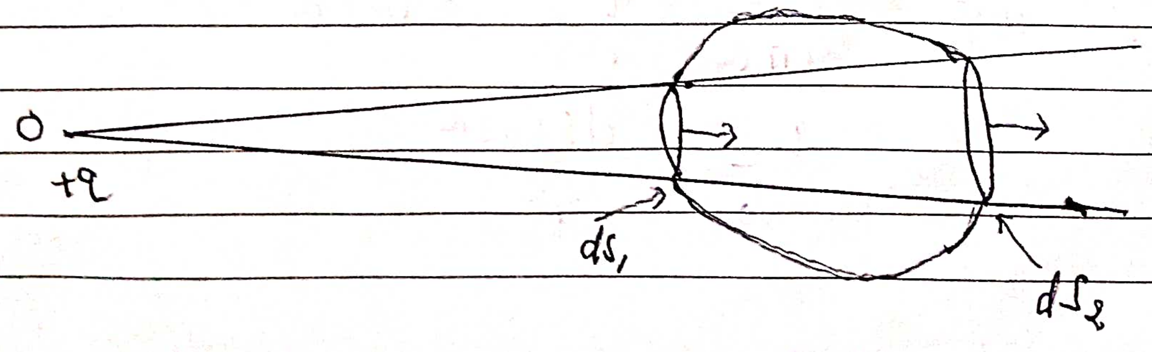
$$\boxed{\therefore \phi = \frac{q}{\epsilon_0}}$$ proved

If there are several charges $+q_1, +q_2, +q_3, \ldots$ etc the same argument can be applied to each in turn. Hence the total flux through the surface due to all of them is

$$\phi = \frac{1}{\epsilon_0}(q_1 + q_2 + q_3 + \ldots) = \frac{1}{\epsilon_0}\Sigma q$$

or $\boxed{\phi = \frac{\Sigma q}{\epsilon_0}}$

If the charge q be outside the surface, the total flux through the surface is zero because the cone with vertex q cuts off area dS_1 when it enters, and area dS_2 where it leaves the surface. The flux through dS_1 is $\frac{q}{4\pi\epsilon_0}d\omega$ inwards and that through dS_2 is $\frac{q}{4\pi\epsilon_0} \cdot d\omega$ outward.



Hence the total flux the two area is zero. This is true for two areas cut off by any such cone. Hence for the whole closed surface the total flux is zero.